ANDHRA UNIVERSITY

SCHOOL OF DISTANCE EDUCATION

ASSIGNMENT QUESTION PAPER SUPPLEMENTARY

M.A. / M.Sc. Mathematics (Final)

COMPLEX ANALYSIS

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) For a > 1 show that $\int_0^{\pi} \frac{d\theta}{a+b\cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$.

   (b) State and prove Residue theorem.

2. (a) State and prove Cauchy's integral formula (first version).

   (b) Show that a Mobius transformation takes circles onto circles.

3. (a) State and prove Mittag - Leffler's theorem.

   (b) State and prove Schwarz reflection principle.

4. (a) State and prove Montel's theorem.

   (b) State and prove Schwartz reflection principle.

Section - B **(4 x 1 = 4)**

5. **Answer all the following :**

   (a) Define analytic function and Mobius transformation.

   (b) Define analytic function and Cauchy - Riemann equations.

   (c) State and prove Schwarz's lemma.

   (d) State and prove Mean - value theorem.

ANDHRA UNIVERSITY
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ASSIGNMENT QUESTION PAPER SUPPLEMENTARY
M.A. / M.Sc. Mathematics (Final)
MEASURE THEORY AND FUNCTIONAL ANALYSIS

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) Show that the outer measure of an interval is its length.
   (b) State and prove bounded convergence theorem.

2. (a) State and prove Radon - Nikodym theorem.
   (b) State and prove Riesz Representation theorem.

3. (a) State and prove Hahn - Banach theorem.
   (b) State and prove the closed graph theorem.

4. (a) State and prove Bessel's inequality.
   (b) Let H be a Hilbert space, and let f be an arbitrary function in H^*. Then prove that there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Prove that m^* is translation invariant.

(b) Let ν be a signed measure on the measurable space $(X, \mathcal{B})$. Suppose E is a measurable set such that $0 < \nu(E) < \infty$. Then prove that there is a positive set A contained in E with $\nu(A) > 0$.

(c) Prove that in a normed linear space, addition and scalar multiplication are jointly continuous.

(d) If x and y are any two vectors in a Hilbert space, then show that $|(x, y)| \leq ||x||.||y||$.

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ASSIGNMENT QUESTION PAPER SUPPLEMENTARY
M.A. / M.Sc. Mathematics (Final)
NUMBER THEORY

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) Let f be multiplicative. Then prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n).f(n)$ for all $n \geq 1$.
   (b) State and prove Euler's Summation formula.

2. (a) For $n \geq 1$ prove that $\phi(n) = \prod_{p|n}\left(1 - \frac{1}{p}\right)$.
   (b) If both g and f*g are multiplicative, then prove that f is also multiplicative.

3. (a) Prove that there are infinitely many primes of the form 4n+1.
   (b) State and prove Euler's Criterion.

4. (a) State and prove quadratic reciprocity law.
   (b) State and prove Euler's criterion.

Section – B **(4 x 1 = 4)**

5. Answer all the following :
   (a) State and Prove Euler - Fermet theorem.
   (b) If $n \geq 1$ then prove that $\log n = \sum_{d|n} \Lambda(\text{d})$.
   (c) State and prove Gauss Lemma.
   (d) Show that there are infinitely many primes of the form 4n-1.

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ASSIGNMENT QUESTION PAPER SUPPLEMENTARY
M.A. / M.Sc. Mathematics (Final)
LATTICE THEORY

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) Define partial ordered set and order isomorphism. Prove that two finite partially ordered sets can be represented by the same diagram if and only if they are order isomorphic.

   (b) Define a weakly complemented lattice and a semi complemented lattice. Prove that every uniquely complemented lattice is weakly complemented.

2. (a) Show that every element of a compactly generated lattice can be represented as a meet of a completely meet irreducible elements.

   (b) Show that the dual, every sublattice and every homomorphic image of a modular lattice is modular.

3. (a) Show that in a Boolean algebra B, a proper ideal M is prime if and only if it is maximal.

   (b) Show that in a section complemented lattice, every ideal constitutes the kernel of at most one congruence relation.

4. (a) Show that every complete Boolean algebra is infinitely distributive.

   (b) Let L be a lattice. Then prove that L is distributive if and only if for any $x \neq y \in L$, there is a prime ideal containing one of then but not both.

Section - B **(4 x 1 = 4)**

5. **Answer all the following :**

   (a) Show that any interval of a lattice is a sub lattice.

   (b) Prove that any Boolean ring can be regarded as a Boolean algebra and Vice - Versa.

   (c) Write about Boolean algebras and Boolean rings.

   (d) Write about congruence relation of lattices.

ANDHRA UNIVERSITY
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ASSIGNMENT QUESTION PAPER SUPPLEMENTARY
M.A. / M.Sc. Mathematics (Final)
LINEAR PROGRAMMING AND GAME THEORY

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) Show that for any matrix A, the row rank and column rank are equal.

   (b) If c is a finite cone, then show that $c^{**} = c$.

2. (a) Define feasible solution, optimal solution of a standard maximization problem and state and prove optimality criterion for standard maximization problem.

   (b) Solve the game with pay off matrix $\begin{bmatrix} -5 & 5 & 0 & -1 & 8 \\ 8 & -4 & -1 & 6 & -5 \end{bmatrix}$.

3. Find $y = (n_1, n_2, \ldots, n_5) \geq 0$ which minimizes $n_1 + 6n_2 - 7n_3 + n_4 + 5n_5$ subject to $5n_1 - 4n_2 + 13n_3 - 2n_4 + n_5 = 20$, $n_1 - n_2 + 5n_3 - n_4 + n_5 = 8$.

4. (a) Show that a game Γ has at most one value.

   (b) Show that the dual problems (A, b, c) have solutions if and only if $\Gamma(A, b, c)$ has an optimal strategy $I = (\partial_0, \partial_1, \ldots, \partial_{m+n})$ with $\partial > 0$. In this case show that there is a One - to - one correspondence between such strategies and solutions of (A, b, c).

Section - B **(4 x 1 = 4)**

5. **Answer all the following :**

   (a) Write about the optimality criterion for simplex method.

   (b) Explain the concept of maximum flow problem.

   (c) Define (i) Standard maximum problem (ii) Canonical maximum problem

   (d) Define (i) convex set (ii) convex hull (iii) convex polytope.

ANDHRA UNIVERSITY
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ASSIGNMENT QUESTION PAPER SUPPLEMENTARY
M.A. / M.Sc. Mathematics (Final)
UNIVERSAL ALGEBRA

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) Prove that L is non distributive lattice iff M_5 or N_5 can be embedded into L.
   (b) If A is congruence permutable, then show that A is congruence modular.

2. (a) Let P be a poset such that $\wedge A$ exists for every subset A of P or such that $\vee A$ exists for every subset of A of P. Then prove that P is a complete lattice.
   (b) Prove that every algebraic lattice is isomorphic to the lattice of closed subsets of some set A with algebraic closure operator C.

3. (a) If A is congruence permutable, then show that A is congruence modular.
   (b) Let $\alpha: A \rightarrow B$ be a homomorphism. Then prove that $Ker(\alpha)$ is a congruence on A.

4. (a) Show that if L is a sub directly irreducible distributive lattice then $|L| \leq 2$.
   (b) State and prove Stone Duality theorem.

Section – B **(4 x 1 = 4)**

5. Answer all the following :
   (a) Define a modular lattice. Show that every distributive lattice is a modular lattice.
   (b) Define the terms (i) Lattice (ii) Partially ordered set.
   (c) State and prove Second isomorphism theorem.
   (d) Let X be a set. Then show that $S \cup (X) \cong 2^X$.

ANDHRA UNIVERSITY
SCHOOL OF DISTANCE EDUCATION
ASSIGNMENT QUESTION PAPER SUPPLEMENTARY
M.A. / M.Sc. Mathematics (Final)

INTEGRAL EQUATIONS

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) Solve the following symmetric integral equation with the help of Hilbert - Schmidt theorem.

   $y(x) = 1 + \lambda \int_0^{\pi} \cos(x+t) y(t) dt.$

   (b) Solve $y'(t) = t + \int_0^1 y(t-x) \cos x \, dx, \ \ y(0) = 4.$

2. (a) State and prove Hilbert Schmidt Theorem.

   (b) Transform the initial value problem $\frac{d^2y}{dx^2} + y = \cos x, y(0) = 0, \ y'(0) = 1$ into an integral equation.

3. (a) Using Green's function, Solve the boundary value problem $y'' - y = x, y(0) = y(1) = 0.$

   (b) Solve $y(x) = \cos x - x - 2 + \int_0^x (t-x) y(t) dt.$

4. (a) Find the eigen values and eigen functions of the homogeneous integral equation

   $y(x) = \lambda \int_1^2 \left(xt + \frac{1}{xt}\right) y(t) dt.$

   (b) Find first and second approximations in the iterative solution of the integral equation

   $\int_0^1 (x+y)^{\frac{1}{2}} [\varphi(y)]^{\frac{1}{2}} \, dy = \varphi(x).$

Section - B **(4 x 1 = 4)**

5. **Answer all the following :**

   (a) Define Fredholm integral equation of the first and second kind.

   (b) Write the four properties to construct the Green's functions.

   (c) Describe the shop stocking problem.

   (d) Solve $y(x) = 1 + \int_0^x y(t) dt.$

ANDHRA UNIVERSITY
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ASSIGNMENT QUESTION PAPER SUPPLEMENTARY
M.A. / M.Sc. Mathematics (Final)
COMMUTATIVE ALGEBRA

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) Prove that the set R of all nilpotent elements in a ring A is an ideal, and A/R has no nilpotent element $\neq 0$.
   (b) State and prove Nakayama's lemma.

2. (a) State and prove second uniqueness theorem.
   (b) State and prove first uniqueness theorem.

3. (a) Prove that the length l(M) is an additive function on the class of A- modules of finite length.
   (b) State and prove going down theorem.

4. (a) State and prove Hilbert basis theorem.
   (b) Prove that in a Noetherian ring every irreducible ideal is primary.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Show that $S^{-1}(A)$ is a flat A- module.
(b) Show that M has a composition series if and only if M satisfies both chain conditions.
(c) Prove that in a Noetherian ring A, every ideal a contains a power of its radical.
(d) If I = r(I), then prove that I has no embedded prime ideals.

ANDHRA UNIVERSITY
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ASSIGNMENT QUESTION PAPER SUPPLEMENTARY
M.A. / M.Sc. Mathematics (Final)
NUMERICAL ANALYSIS AND COMPUTER TECHNIQUES

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1 (a) Write the difference between FUNCTION and SUBROUTINE.

(b) Give the computed value of the following FORTRAN EXPRESSION.
Using I = -2, J = 5, K = -2, A = 10.5, B = 2.5.
i). J ** I / J * K ii). A ** I – B .

2. (a) Write a Fortran program to evaluate the integral $\int_1^2 \sin 2x \, dx$ using simpson's rule with 4 sub intervals.

(b) Write a FORTRAN program to find the transpose of a 3X3 matrix.

3. (a) Solve the boundary value problem $u'' = u + x, u(0) = 0, u(1) = 0$ with $h = \frac{1}{4}$ use the numerov method.

(b) Evaluate the integral $I = \int_{-1}^{1} (1 - x^2)^{\frac{3}{2}} \cos x \, dx$ by using Gauss Legendre 3 point formula.

4. (a) Solve the initial value problem $u' = -2tu^2, u(0) = 1$ with $h = 0.2$ on the interval [0,1] . Use the fourth order classical Runge - Kutta Method.

(b) Using the following data , find $f'(6.0)$.

x	6.0	6.1	6.2	6.3	6.4
f(x)	0.1750	-0.1998	-0.2223	-0.2422	-0.2596

Section - B **(4 x 1 = 4)**

5. **Answer all the following :**

(a) Write about the Variable with examples.
(b) Explain shooting method.
(c) Write about single step method.
(d) Explain the third order Runge - Kutta method.